Envelope based nonlinear blind deconvolution approach for ultrasound imaging

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***Abstract*—The resolution of ultrasound medical images is yet an important problem despite of the researchers efforts. In this paper we presents a nonlinear blind deconvolution to eliminate the blurring effect based on the measured radio-frequency signal envelope. This algorithm is executed in two steps. Firslty we make an estimation for Point Spread Function (PSF) and, secondly we use the estimated PSF to remove, iteratively their effect. The proposed algorithm is a greedy algorithm, called also matching pursuit or CLEAN. The use of this algorithm is motivated beacause theorically it avoid the so called inverse problem, which usually needs regularization to obtain an optimal solution. The results are presented using 1D simulated signals in term of visual evaluation and nMSE in comparison with the two most kwown regularisation solution methods for least square problem, Thikonov regularization or l2-norm and Total Variation or l1 norm.**

***Keywords*—ultrasound, resolution improvement, nonlinear blind deconvolution, matching pursuit.**

I. Introduction

Ultrasound imaging is a diagnostic method widely used in the clinical investigations. Despite of its safety and other advantages, it has a great disadvantage. The resolution is degraded some effects like attenuations, refractions, nonlinearities, frequency choose or probe properties. For example, the wave propagation in the tissues produces nonlinearities, which affect the central frequency of the pulse. Also, the properties of the piezoelectric crystal - the limited bandwidth and the pulse shape – produce a blurring and a rejection of high frequencies in final signals/images [1], [2].

The resolution improvement can be made in two directions. Firstly by improving the imaging systems, which generates very high prices for final products and, secondly using post processing signals. This second approach is more practical in many cases because it requires only a computation system and some algorithms and can offer interesting results.

In ultrasound imaging, the obtained B-mode image supposes the interaction between the acoustic beam, generated by the transducer and, the scanned tissues. Usually, the phenomena are not linear but, for computations simplicity, the greatest part of the methods proposed in literature suppose that the acquired signal is a linear combination between the reflectivity function (*i.e.* the scanned environment) and the pulse. This can be written in mathematical formulation, as follows:

$$y(t) = x(t) \otimes h(t) + n(t). \quad (1)$$

where $\otimes$ is the convolution operator, $y(t)$ is the measured signal, $h(t)$ is the system impulse response, $x(t)$ is reflectivity function and, $n(t)$ is a Gaussian white noise.

First studies proposed in the literature used a measured Point Spread Function (PSF) [3], [4]. The use of an only one PSF to deconvolve the entire image is not feasible because of the attenuations, reflections, refractions and other processes which change the shape of the PSF. A solution is to estimate the PSF from the acquired signals by supposing that it is slow variant in time. This follows to divide the image in segments were ones considers that it is constant and estimates a PSF for each segment. These methods are blind deconvolution methods and, the problematic of PSF estimation is made using different assumptions.

An approach proposes to estimate the PSF using high order statistics and the problem was solved using 1D implementation [5]. The method offers interesting results, but the implementation and computation time made it very difficult to implement in high dimensional space.

Taxt et al. introduced and improved a new method of PSF estimation using the Cepstrum and Homomorphic Deconvolution properties [6][7][8][9]. In this approach it is considered that the PSF spectrum is a function more smooth than the reflectivity function. The restoration was made supposing that the reflectivity function has a Gaussian distribution, so the Wiener filter was used. Also, the proposed procedures use the RF signals or their envelope.

A new approach of PSF estimation was introduced in [10]. Here the PSF is estimated by supposing that reflectivity function has a spectrum similar to white noise and the authors proposed a de-noising procedure to eliminate it [11]. This procedure uses also the Homomorphic Deconvolution to separate and to eliminate the reflectivity function component from the acquired signal [12]. An outlier resistant de-noising procedure was proposed in the de-noising procedure [13]. The sparsity assumption for tissue shape was proposed in [14], a procedure which uses the envelope of the acquired image. Also, a hybrid parameterization of inverse filter with sparsity assumptions is proposed by O. Michailovich in [15].

Other kinds of approaches were proposed in [16] or [17]. In [16] the authors proposed and expectation-maximization algorithm that solved the problem iteratively alternating between Wiener filtering and wavelet-based de-noising. The PSF is considered *a priori* known. In [17], 2-steps deconvolution algorithm was proposed, where in first step the PSF is estimated using the Cepstrum technique and for deconvolution a two steps iterative/thresholding (TwIST) is used.

In this paper we want to present an idea of a deconvolution algorithm which intends to improve the ultrasound signals using a greedy algorithm, similar to matching pursuit [18] or CLEAN algorithm in radio-astronomy [19]. The presented algorithm use the RF signal envelope because is difficult to estimate the central frequency of PSF. This fact is due because of nonlinearities presented in the wave propagation in tissues.

It is well known that in ultrasound imaging a part of the generated pulse is reflected when it find an interface between two tissues with different physical properties. Using this information, we suppose that the reflectivity function has a Laplace probability of density function (PDF) as in [10], [14], [15] or [17].

The present paper is organized as follows: in Section I was made a short presentation of the field and the most important signal processing approaches for resolution improvement, Section II presents the used methods for this algorithm and comparative algorithms, Section III describes the simulations, Section IV shows the results and, Section V concludes the current study.

II. METHODS

In the following are presented the most important methods used in this algorithm, as follows: Hilbert Transform for envelope extraction, Homomorphic deconvolution and soft-thresholding denoising for PSF estimation, and matching pursuit algorithm for nonlinear time domain deconvolution (Subsection II.A).

For results evaluation was implemented Thikhonov regularization and Total Variation Regularization. These methods were presented in Subsection II.B.

A. Proposed method

The proposed method started from the acquired RF signals. Using these signals we extracted the envelope and afterwards, this envelope was used in two step blind deconvolution algorithm. The algorithm was a two steps algorithm because in the first step the PSF was extracted, and then, this PSF was used in the deconvolution algorithm. Figure 1 presented the diagram of the proposed algorithm.

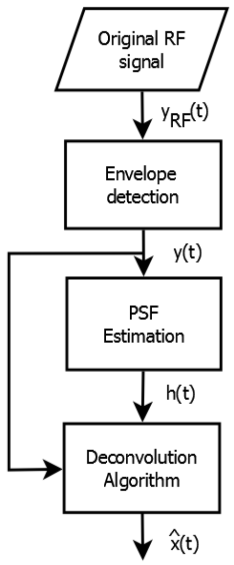

Figure 1. The steps of the algorithm implementation.

1) Hilbert transform

In mathematics and in signal processing, the Hilbert transform was a linear operator which taken a function $y_{RF}(t)$, and produced a function $H(y_{RF})(t)$, with the same domain. In signal processing, it was widely used to derive the analytic representation of a signal $y_{RF}(t)$, as follows:

$$y_a(t) = y_{RF}(t) + jH(y_{RF})(t). \qquad (2)$$

where $y_a(t)$ was the analytic signal, $y_{RF}(t)$ was the original signal and, $H(y_{RF})(t)$ was the Hilbert Transform of $y_{RF}(t)$.

In the situation when the signal $y_a(t)$ was a sinusoidal signal, by applying the absolute value operator we obtained its envelope, as follows:

$$y(t) = |y_a(t)|. \qquad (3)$$

where $y(t)$ was the signal envelope, $y_a(t)$ was the same analytic signal and $|\cdot|$ was the absolute value operator.

2) Point Spread Function estimation

The main idea in the ultrasound pulse estimation was that it was a smooth function and a reflectivity function had a wide and more uniform spectrum. Using this assumption one can change the signals separation problems in a de-noising one. For this we used the logarithm and homomorphic deconvolution properties.

If we applied the logarithm to the left and right members of the Fourier Transform of the equation (1) we obtained the sum of the input signals [12]:

$$\log(Y(\omega)) = \log(H(\omega)) + \log(X(\omega)). \qquad (4)$$

where *log* was the natural logarithm and, $Y(\omega)$, $H(\omega)$ and $X(\omega)$ were the Fourier Transform for $y(t)$, $h(t)$ and, respectively, $x(t)$. The noise parameter was removed in (4) for computation simplicity. Then we transformed the input signal into a linear operation. This could discriminate between the signals using the above presented assumptions that PSF was a much smooth function and the wave separation problem could be changed in a de-noising one. The algorithm was proposed in [10]. The main idea of this technique was the use of a de-noising method in the frequency domain by applying a wavelet soft thresholding and an outlier resistant de-noising algorithm. The threshold was calculated using the formula [20]:

$$T = \sigma\sqrt{2\log_e(N)}. \quad (5)$$

where N was the length of the array, log_e was natural logarithm and, σ was the noise variance. The σ parameter was automatically estimated with the formula:

$$\sigma = M_x / 0.6745. \quad (6)$$

where M_x was the median absolute value of the finest decomposition level.

3) Deconvolution algorithm

The presented algorithm for deconvolution was inspired by the realization of the acquired ultrasound wave, *i.e.* the resulted signal was the superposition of the reflected pulses from human body. Using the assumption that reflectivity function was a sparse function (*i.e.* had a Laplace PDF) we wanted to extract iteratively, from the envelope of the measured signal the influence of the most important blurred scatter and to replace it with a Dirac pulse, at the same position, in a signal with all positions zero in the beginning.

The algorithm was a greedy algorithm (Matching Pursuit or CLEAN) because it worked top-down: it made a locally optimal choice in hope that solving the sub-problem, at the end, the final solution was optimal [21].

In our approach we considered that the locally optimal choice was the strongest reflector because its amplitude could hide the neighbor reflectors with lower amplitude.

So, we set the locally optimal problem the iterative extraction of the highest blurred scatter from the residual signal in hope that the final result contained a highest possible number of scatters.

The deconvolution algorithm was synthesized in the following:

Input: signal $y(t)$, PSF $h(t)$.
Output: reflectivity function $\hat{x}(t)$.
Initialisation:
$x(t) \leftarrow 0$; $R_0(t) \leftarrow y(t)$; $aux(t) \leftarrow 0$, $n \leftarrow 0$.
Repeat
- find t_i and $A(t_i)$ where $A(t_i) \leftarrow \max\{R(t) | t_i \in t\}$;
- $x(t_i) \leftarrow A(t_i)$;
- $aux(t) \leftarrow A(t_i)$;
- $R_{n+1}(t) \leftarrow R_n(t) - h(t) \otimes aux(t)$;
- $aux(t) \leftarrow 0$;
- $n \leftarrow n+1$;

Until stop criterion ($R_n(t) < threshold$).

Here, $R(t)$ was also called residual signal, $A(t_i)$ was the value of maximum amplitude at the position t_i and $\otimes$ was the convolution operator. The threshold was a value which usually was set manually.

The algorithm had also, another advantage – it was a time domain deconvolution. This made it to avoid the *inverse problem* in signal processing, which was well known as one of the difficult problems in signal processing.

B. Comparative methods

Generally, in the inverse problem the natural solution for resolving the system was to find the minimum solution for the least square criterion

$$\min\{\| \mathbf{H}x - y \|_2^2\} \quad (7)$$

where $\mathbf{H}$ was the Toeplitz matrix of the estimated 1D PSF and $\|\cdot\|$ was the Euclidian norm. The problem was that usually the $\mathbf{H}$ matrix was not invertible and then, the solution was not unique.

1) Tikhonov Regularization

The Tikhonov Regularization (TkR) was a regularization method to penalize the least square solution for the ill posed or ill conditioned problems in Hadamard sense [22], [23]. Tikhonov introduced in the least square solution a regularization parameter to minimize the size of the solution. The general form was:

$$\min\{\| \mathbf{H\Gamma} - y \|_2^2 - \| \quad x \|^2\}. \quad (8)$$

where $\mathbf{\Gamma}$ was the Tikhonov matrix, $\|\mathbf{H}x - y\|_2^2$ was the residual norm and $\| \mathbf{\Gamma} x \|^2$ was the solution norm. The Tikhonov matrix was usually $\mathbf{\Gamma} = \alpha\mathbf{I}$ where the $\mathbf{I}$ matrix was the unit matrix and the α parameter was called regularization factor. The optimal value for α was find with the L-curve method, which was a log-log representation between residual and solution norms [24].

2) Total Variation Regularization

Total Variation was similar to (8) with the difference that the l_2-norm was replaced by l_1-norm [25]. The Equation (8) became to minimize:

$$\min\{\| \mathbf{H}x - y \|_2^2 - \alpha\|x\|_1\}. \quad (9)$$

where the parameters had the same signification and the l_1-norm was $\|x\|_1 = \sum x_i$ with $i = \overline{1, N}$ and N the length of reflectivity function x. In comparison with Tikhonov regularization, this algorithm was not linear. To solve Total Variation usually was used iterative algorithms to find optimal solution, like Quasi-Newton methods [26].

III. SIMULATIONS

For the simulations we used sparse synthetic signals contaminated with the Gaussian white noise to simulate reflectivity function. The length of the signals was 512 points the sampling frequency was 20 MHz and the central transducer frequency was 3.2 MHz. This corresponded to sequence of 160 μs and an approximately 3.94 cm deep scanning (for a standard ultrasound velocity c = 1540 m/s).

The synthetic RF signals were made using the circular convolution of the generated reflectivity function and an ideal PSF consisted of a sinusoidal signal with frequency of 3.2 MHz type multiplied by a Gaussian envelope.

The reflectivity function was realized by a random generated sparse signal which added a Gaussian noise corresponding to different SNR noises. With this noise we intended to simulate different types of tissues. For example, we find more speckle noise and weak scatters in the soft tissues, like abdominal tissues.

In the Figure 2 was presented the generation of simulated RF signals. The result was made following the equation (1), *i.e.* the convolution between the reflectivity function and the PSF.

The simulations were performed in MATLAB, using a PC with Intel i5 processor and 4 GB of RAM.

The sparse signals and the noise have been generated randomly with the *sprandn* and *randn* functions.

For wavelet decomposition and de-noising we used Wavelab Toolbox, downloaded from *http://www-stat.stanford.edu /~wavelab/.*

To test the proposed algorithm, we got the RF signal, we obtained its envelope using the Hilbert transform. From the envelope we estimated the PSF using the procedure presented in Section II.A.2). At the end, we used the PSF to reconstruct the reflectivity function with different approaches.

IV. RESULTS AND DISCUTIONS

The results were focused to evaluate the ability of the proposed algorithm to reconstruct the reflectivity function starting from the acquired RF signal envelope. The results were compared with two important state of the art techniques in signal processing: Tikhonov regularization and Total Variation.

These were presented in terms of visual and quantitative evaluation. For quantitative we measured the execution time for each method and we computed the normalized Mean Square Error (nMSE). The nMSE is defined as follows:

$$nMSE = E\left[\frac{\|\hat{x}-x\|_2^2}{\|x\|_2^2}\right] \tag{10}$$

where E is the statistical expectation, x is the original reflectivity function, $\hat{x}$ is the resulted reflectivity function and $\|\cdot\|$ is the l_2-norm.

Figure 3 presented the simulation results in terms of visual evaluation. We can see that our algorithm overcome the dynamic algorithms. In the top was presented the original reflectivity function, which we wanted to extract. Then, the results of with Tikhonov regularization had many oscillations, which was normal because Tikhonov regularization is usually used when the resulted signal has a Gaussian distribution. The second used algorithm (TV) offered a sparse solution which is more similar to the desired signal, but not enough.

In the Table I we presented the quantitative evaluations for the presented methods. In terms of nMSE we observed that the results confirmed the above presented

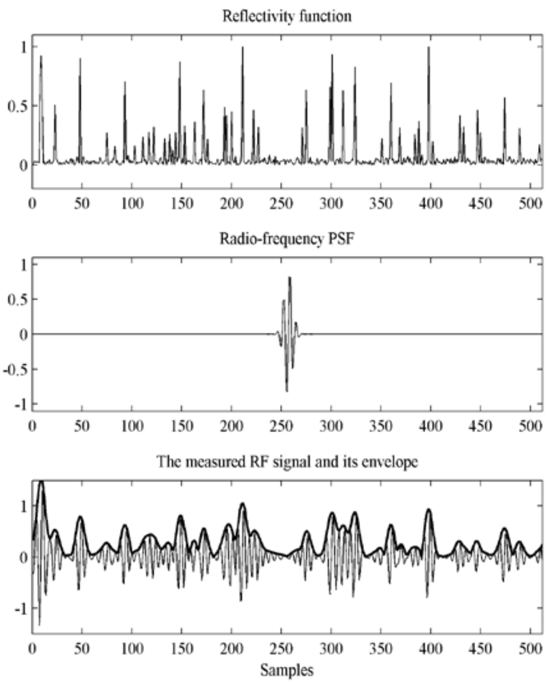

Figure 2. Simulated signals. Top: the generated reflectivity function. Middle: the generated PSF. Bottom: the resulted RF signal and its envelope.

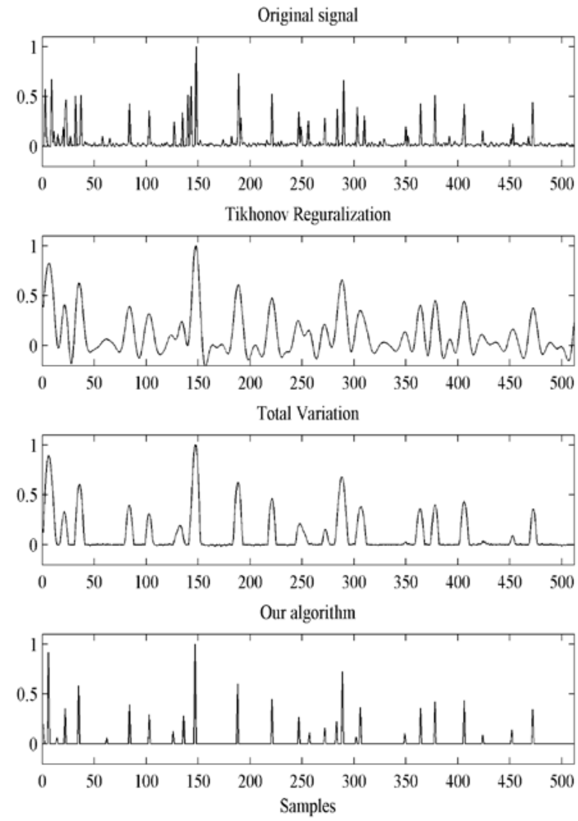

Figure 3. Top: original reflectivity function; Middle-top: results obtained with TkR; Middle-bottom: results obtained with TV; Bottom: results obtained with our algorithm.

TABLE I.
QUANTITATIVE EVALUATION FOR TESTED ALGORITHMS

Evaluation Parameter	TkR	TV	Our. Alg.
Time[s]	0.1664	13.6232	0.0652
nMSE	4.0180	3.3083	0.7399

visual results. Our algorithm had also, good results in execution time. Algorithm speed offered an expected result compared with the TV because TV is nonlinear and it used an iterative Newton algorithm, but compared with TkR is surprisingly because it is solved without iterative procedures.

V. CONCLUSIONS AND FUTURE WORK

This paper presented a "greedy" blind blind deconvolution algorithm which intended to extract the reflectivity function from the envelope of acquired RF signals.

Using simulated signals, the method presented better results in term of visual and quantitative evaluation (nMSE). Also, the method was faster than classical dynamic programming algorithms.

Like future work, we want to test the proposed algorithm onto real ultrasound images in booth 1D and 2D implementation.

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